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CHAP:3: KINETIC THEORY OF GASES AND RADIATION

Pressure P, volume V & temp. T of gas are three variables of gas.

Boyle's law: $V \propto \frac{1}{P}$ at constant T.

Charles law: $V \propto T$ at constant P

Gay Lussac's law: $P \propto T$ at constant V.

All these three laws apply to fixed mass(m) of an enclosed gas.

Combining these three laws into a single relation for a fixed mass of gas yields ideal gas equation.

$PV \propto T$ or $PV = nRT$ —(1) [n = no. of moles, R = Universal gas constant, R = 8.314 J/mol K]

$\therefore \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ —(2)

$n = \frac{\text{mass of gas } (M)}{\text{molar mass } (M_0)} = \frac{N}{N_A}$ —(3)

where N = No. of molecules in a gas

N_A = Avogadros number (No. of molecules in 1 mole of gas).

Also $PV = Nk_BT$ —(4) --(k_B = Boltzmann constant)

From eq^n (1) & (4) $R = N_Ak_B$ —(5)

The gas obeying the eq^n of state $PV = nRT$ at all pressures & temp. is called as an Ideal gas.

The above eq^ns & laws are strictly valid for real gas at only low pressure & high temp (near to liquefaction temp. of the gas).

* Ideal Gas and Real gas:

- Ideal Gas: 1) It obeys ideal gas eq^n (Eq^n of state) at all pressures and temperature.
2) The intermolecular interactions are absent
3) It has zero molecular size & negligible molecular force
4) The energy possessed by ideal gas molecules is purely translational K.E. & molecules are always in random motion except at absolute zero.
5) Collision bet^n gas molecules & walls of container is perfectly elastic.

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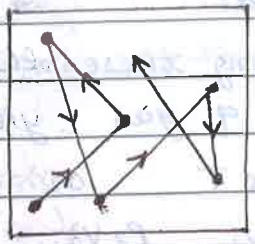
- **Real gas**: 1) Real gases are composed of atoms or molecules which do interact with each other.
2) Intermolecular reactions is absent except at low pressure & high temp^{s}.

* **Mean free path**: The average distance traversed by a gas molecule with constant velocity bet^{n} two successive collisions is called as mean free path.

(a) A gas with molecules dispressed in the container



(b) A typical gas molecules executing random motion.

The molecules of a gas are uniformly dispresed throughout the volume of a gas as shown in fig (a). The molecules executing random motion as shown in fig (b). when a molecule approaches another molecule, there is a repulsive force bet^{n} them. There is elastic collision bet^{n} them. So the speed as well as direction of motion of molecules change abruptly. They also collide with the wall of container. They exerts force on each other during collisions only. Bet^{n} two successive collisions the molecules move along straight line with constant velocity, this straight line dist^{n} is called as free path. The average dist^{n} bet^{n} two successive collisions is called as mean free path.

The mean free path is given by

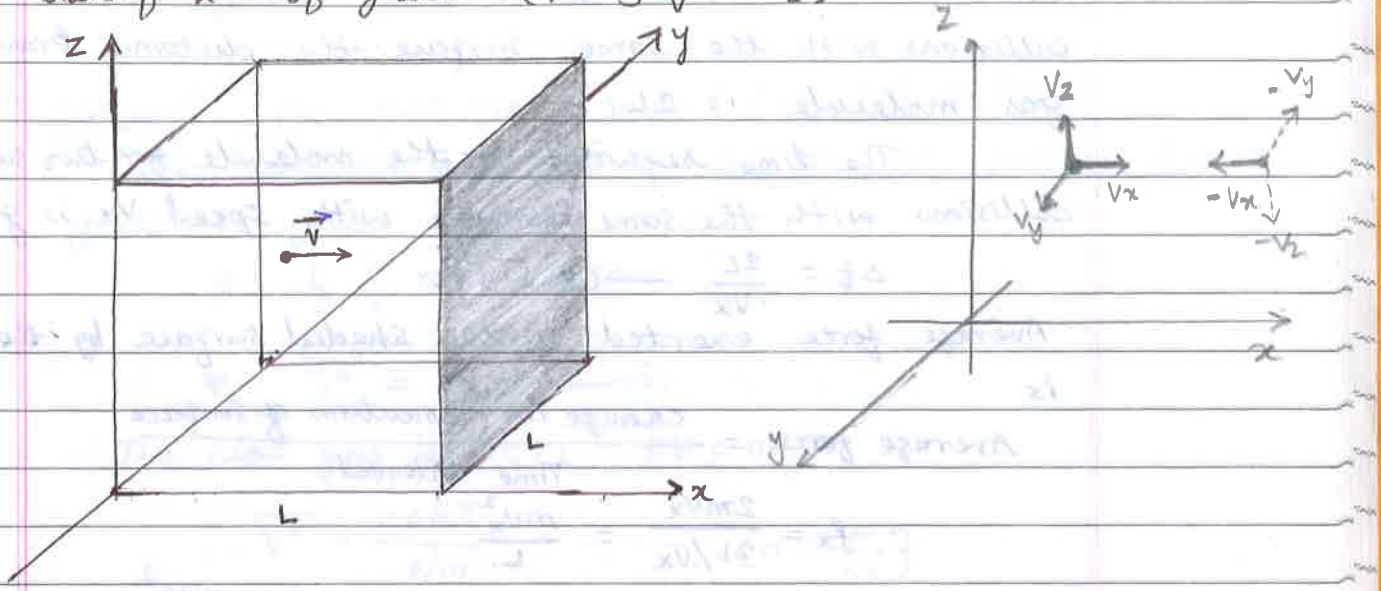
$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 \rho} \quad --- \quad (\rho = N/V)$$

$$\therefore \lambda = \frac{1}{\sqrt{2}\,\pi d^2 (N/V)}$$

where d = diameter of a molecule
ρ = density of gas
V = volume of gas
N = No. of molecules of gas.

This eq? shows that λ is inversely proportional to the size of the molecule (diameter d), smaller the size of molecule, less is the chance for collision & larger is the mean free path. λ is also inversely proportion to density of gas.

Q. Obtain an expression of pressure of an ideal gas on the basis of k.T. of gases. $\left(P = \frac{1}{3}\frac{N}{V} m \overline{v_2}\right)$



Consider 'n' mole of an ideal gas enclosed in a cubical box of volume $V = L^3$, with sides of the box parallel to the coordinate axes. Gas consist of 'N' no of molecules each of mass 'm'. The molecules are continuously moving randomly in all possible directions. They collide with each other as well as with the walls of container & bouncing back.

Consider a molecule moving with velocity $\vec{V}$ towards shaded wall of the cube (wall parallel to yz plane). $\vec{V}$ resolved into 3 components V_x, V_y & V_z along x, y & z axis respectively. Let the molecule moving with velocity V_x towards shaded wall keeping y & z components constant.

$\therefore$ Initial momentum of gas molecule $= mV_x$ —①

This molecule collide with the surface & rebounce with speed $-V_x$. $\therefore$ After collision.

Final momentum of gas molecule $= -mV_x$ —②

$\therefore$ change in momentum of gas molecule = Final momentum − Initial momentum

$$\Delta P_x = -mV_x - mV_x$$

$$\Delta P_x = -2mV_x \text{ ——— ③}$$

-ve sign is due to loss of momentum by gas molecule, This momentum is gain by shaded surface.

∴ Change in momentum of surface $= +2mv_x$ —— (4)

After collision with shaded surface, molecule travel towards opposite surface & again return back & make 2nd collision with the shaded surface, during two successive collisions with the same surface. the distance travel by gas molecule is 2L.

The time required for the molecule for two successive collisions with the same surface with speed v_x is given by

$$\Delta t = \frac{2L}{v_x} \quad \text{—— (5)}$$

Average force exerted on the shaded surface by the molecule is

$$\text{Average force} = \frac{\text{change in momentum of surface}}{\text{Time interval}}$$

$$\therefore f_x = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

The force exerted by 1st molecule is $f_{x_1} = \frac{mv_{x_1}^2}{L}$

similarly by 2, 3, 4 —— molecules with velocities $v_{x_2}, v_{x_3}, v_{x_4}$ —— v_{x_N}. So average force on the wall is.

$$F_x = \frac{m}{L}\left(v_{x_1}^2 + v_{x_2}^2 + \text{———} + v_{xN}^2\right) \quad \text{—— (6)}$$

The average pressure by the molecules is

$$p = \frac{\text{Average force}}{\text{Area of shaded wall}} \quad \text{———} \quad \left(p = \frac{F}{A} = \frac{F}{L^2}\right)$$

$$\therefore p = \frac{m\left(v_{x_1}^2 + v_{x_2}^2 + \text{———} + v_{xN}^2\right)}{L^3} \quad \text{——} \quad (L^3 = V)$$

$$\therefore p = \frac{m\left(v_{x_1}^2 + v_{x_2}^2 + \text{———} + v_{xN}^2\right)}{V} \quad \text{— (7)}$$

The average of the square of x components of velocities are

$$\overline{v_x^2} = \frac{v_{x_1}^2 + v_{x_2}^2 + \text{———} + v_{xN}^2}{N} \quad \text{— (8)}$$

∴ From eq^n (7) & (8)

$$p = \frac{mN\overline{v_x^2}}{V} \quad \text{———} \quad (9)$$

Now for all the molecules along x, y & z axes.

$$\overline{V^2} = \overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}$$

By symmetry $\overline{V_x^2} = \overline{V_y^2} = \overline{V_z^2} = \frac{1}{3}\overline{V^2}$.

∴ From eq^n (9), the pressure is

$$\boxed{P = \frac{1}{3}\frac{N}{V} m \overline{V^2}} \quad \text{—— (10)}.$$

* Expression for Root mean square (rms) Speed:

The pressure exerted by gas on the basis of K.T. of gases is given by.

$$P = \frac{1}{3}\frac{N}{V} m\overline{V^2} \quad \text{—— (1)}$$

$$\therefore \overline{V^2} = \frac{3PV}{Nm} \quad \text{—— (2)}$$

The ideal gas eq^n is $PV = nRT$.

$$\therefore \overline{V^2} = \frac{3nRT}{Nm} \quad \text{---} \quad \left(n = \frac{N}{N_A}\right)$$

$$\therefore \overline{V^2} = \frac{3RT}{N_A m} \quad \text{---} \quad (M_0 = N_A \cdot m) \rightarrow \text{Molecular mass of gas.}$$

$$\therefore \overline{V^2} = \frac{3RT}{M_0}$$

$$\therefore \boxed{\sqrt{V^2} = V_{rms} = \sqrt{\frac{3RT}{M_0}}} \quad \text{—— (3)}$$

For Nitrogen at 300 K, $V_{rms} = 517$ m/s.

For oxygen at 300 k, $V_{rms} = 483$ m/s.

Q. What is the interpretation (relation) of temp. ~~in~~ to the KE of gas molecules?

⟹ The eq^n of pressure exerted by gas on the basis of K.T. of gases

$$P = \frac{1}{3}\frac{N}{V} m \overline{V^2}$$

$$\therefore PV = \frac{2}{3} N \left(\frac{1}{2} m \overline{V^2}\right) \quad \text{—— (1)}$$

where $\frac{1}{2} m\overline{V^2}$ is average ~~KE~~ translational KE of a molecule.

The internal energy of an ideal gas is purely Kinetic

The average total energy E, is given by

$$E = N \cdot \frac{1}{2} m \overline{v^2} \quad \text{---(2)}$$

$$\therefore \quad pV = \frac{2}{3} E \quad \text{---(3)}$$

Using Ideal gas equation

$$pV = N k_B T = \frac{2}{3} E$$

$$\therefore \quad E = \frac{3}{2} N k_B T \quad \text{---(4)}$$

or

$$\frac{E}{N} = \frac{3}{2} k_B T \quad \text{---(5)}$$

This shows that the average energy per molecule is proportional to the absolute temp T of the gas.

This eq^n gives interpretation (relation) of the macroscopic parameter of the gas T to the KE of a molecule

* Show that total translation energy Constribution of the molecule is $\frac{3}{2} k_B T$. [Law of equipartition of energy].

$\Rightarrow$ The KE of a single molecule moving ~~along a strai~~ in a space is

$$KE = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2 \quad \text{---(1)}$$

for a gas at a temp. T, the average KE per molecule is denoted by $\langle KE \rangle$

$$\therefore \langle KE \rangle = \left\langle \frac{1}{2} m v_x^2 \right\rangle + \left\langle \frac{1}{2} m v_y^2 \right\rangle + \left\langle \frac{1}{2} m v_z^2 \right\rangle \quad \text{---(2)}$$

but $\left\langle \frac{1}{2} m v_x^2 \right\rangle = \left\langle \frac{1}{2} m v_y^2 \right\rangle = \left\langle \frac{1}{2} m v_z^2 \right\rangle = \frac{1}{2} k_B T$

$$\therefore \boxed{\langle KE \rangle = \frac{3}{2} k_B T} \quad \text{---(3)}$$

It shows that the total translational energy Contribution of the molecule is $\left(\frac{3}{2} k_B T\right)$.

* Degrees of Freedom :-

1) If a molecule moves along a straight line, then only x-coordinate and only one velocity component V_x will be sufficient to describe its location, so it posses - One degree of freedom
2) If a molecule moves in a plane, then two coordinats x & y & two velocity components V_x & V_y will be sufficient to describe its location; so it posses two degrees of freedom
3) If a molecule moves in a space, then three coordinates x, y & z & three velocity components V_x, V_y & V_z will be sufficient to describe its location; so it posses three degrees of freedom.

∴ Degrees of freedom of a system are defined as the total number of coordinates or independent quantities required to describe the position and configuration of the system completely.

* State the law of equipartition of energy of gas molecules. Find the eqns of their total energy for monoatomic, diatomic For rigid rotator & non rigid rotator molecules.

⟹ **Law of Equipartition of Energy :-**

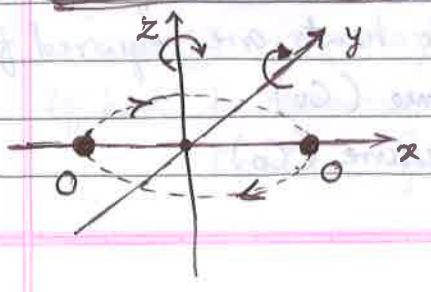
For a gas in thermal equilibrium at a temp. T. the average energy for molecule associated with each quadratic term is $\frac{1}{2} k_B T$.

- **For monoatomic gas** like helium (He), contains He atoms. An He atom has 3 translational degrees of freedom (dOF) so its total energy is purely 3 translational dOF

$$\therefore E = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 = \frac{1}{2}k_BT + \frac{1}{2}k_BT + \frac{1}{2}k_BT$$

$$\therefore \boxed{E = \frac{3}{2}k_BT}$$

- **For diatomic (rigid rotator) gas!** Consider a diatomic gas molecule like O_2 or N_2 which is rigid rotator. These two molecules lying along x-axis. These molecules has 3-translational dOF & in addition 2-rotational dOF along y & z axis



This diatomic molecule can rotate about its C.M. in two directions. Let to molecular axis

Let I_y & I_z are moment of inertia & w_y & w_z be the respective angular speeds about y & z axis. Its total energy is given by

$$E = \left(\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2\right) + \left(\frac{1}{2}I_y w_y^2 + \frac{1}{2}I_z w_z^2\right)$$

$$E = \frac{3}{2}k_B T + \left(\frac{1}{2}k_B T + \frac{1}{2}k_B T\right)$$

$$E = \frac{3}{2}k_B T + k_B T$$

$$\boxed{E = \frac{5}{2}k_B T}$$

- For diatomic (non rigid rotator) gas:

For the diatomic non rigid rotator gas molecules like O_2, N_2, CO. can oscillate along the internuclear axis only. so the molecules possess 3-translational, 2-rotational and one vibrational (with two components) dOF.

So total energy is given by

$$E = E_{(Translational)} + E_{(Rotational)} + E_{(vibrational)}$$

$$E = \left(\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2\right) + \left(\frac{1}{2}I_y w_y^2 + \frac{1}{2}I_z w_z^2\right) + \left(\frac{1}{2}mu^2 + \frac{1}{2}kx^2\right)$$

$$E = \frac{3}{2}(k_B T) + 2\left(\frac{1}{2}k_B T\right) + 2\left(\frac{1}{2}k_B T\right)$$

$$\boxed{E = \frac{7}{2}k_B T}$$

$\left[KE = \frac{1}{2}mu^2,\ PE = \frac{1}{2}kx^2 \Rightarrow E_{vibrational}\right]$

* Specific heat Capacity :- When the temp of gas is increased with small value, causes considerable change in volume and pressure. So two specific heats are required for gas

1) Specific heat at constant volume (C_v)
2) Specific heat at constant pressure (C_p).

* Mayer's Relation:-

Consider 1 mole of an ideal gas enclosed in a cylinder by light, frictionless airtight piston. Let P, V & T be the pressure, volume & temp. of gas respectively.

If the gas is heated so that its temp. rises by dT, keeping its volume constant, so the amount of heat supplied to the gas (dQ_1) is used to increase the internal energy (dE) of gas, as volume is constant so no work is done

$$\therefore dQ_1 = dE = C_V dT \quad \text{——} (1)$$

where C_V = Molar specific heat of gas at constant volume.

If the gas is heated, so that its temp. rises by dT, keeping its pressure constant, so volume of gas increases by dV. The amount of heat supplied to the gas (dQ_2) is used to increase its internal energy (dE) & to do workdone (dw) to increase its volume. ($dw = PdV$)

$$\therefore dQ_2 = dE + dW = C_p dT \quad \text{——} (2)$$

where C_p = Molar specific heat of gas at constant pressure.

From (1) & (2)

$$C_p dT = C_V dT + dW \quad \text{--- } (dw = PdV)$$

$$C_p dT - C_V dT = PdV$$

$$\therefore (C_p - C_V) dT = PdV \quad \text{——} (3)$$

For 1 mole of gas $PdV = RdT$

$$\therefore (C_p - C_V) dT = RdT$$

$$\therefore \boxed{C_p - C_V = R} \quad \text{——} (4)$$

Eqn (4) is Mayer's relation.

If heat is measured in calories & workdone in joule, then

$$C_p - C_V = \frac{R}{J} \quad \text{--- } (J = \text{Mechanical equivalent of heat})$$

As $S_p = C_p \rightarrow$ Principal sp. heat at constant pressure &

$S_v = C_v \rightarrow$ Principal sp. heat at constant volume.

The Relation bet? Molar and Principal sp. heats is

$C_p = M_0 S_p$ (where M_0 is molar mass of gas)

$C_v = M_0 S_v$

$\therefore M_0 S_p - M_0 S_v = \frac{R}{J}$

$\therefore \boxed{S_p - S_v = \frac{R}{M_0 J}}$ — (5)

a. Find the specific heats of gases for (a) monoatomic (b) diatomic (c) polyatomic gas molecules by law of equipartition of energy & also find the value of ratio of sp-heats (γ).

⟹ (a) Monoatomic gas:-

For monoatomic gas enclosed in a container, at constant temp T. & Containing N_A ~~molecules~~ atoms, each atom has 3 translational dOF, ∴ Average energy per atom is

$\frac{3}{2} k_B T$ — (4)

So total internal energy is

$E = \frac{3}{2} N_A k_B T$ — (1)

The molecular sp heat at constant volume is

$C_V = \frac{dE}{dT} = \frac{3}{2} N_A k_B$ ——— $(R = k_B N_A)$

$\therefore C_V = \frac{3}{2} R$ — (2)

but $C_p - C_V = R$

$\therefore C_p = C_V + R = \frac{3}{2} R + R$

$\therefore C_p = \frac{5}{2} R$ — (3)

$\gamma = \frac{C_p}{C_v} = \frac{5/2 R}{3/2 R}$ $\therefore \boxed{\gamma = \frac{5}{3}}$ — (4)

(b) Diatomic gas (Rigid rotator).

For a gas consist of diatomic molecules such as O_2, N_2, CO, HCl. enclosed in a container at constant temp. T. if treated as rigid rotator, each atom have 3- Translation and 2- Rotational dOF. The internal energy one one mole of gas is

$$E = \frac{3}{2} N_A k_B T + \frac{2}{2} N_A k_B T$$

$$E = \frac{5}{2} N_A k_B T \quad \text{—— (1)}$$

The molar sp. heat at constant volume is

$$C_V = \frac{dE}{dT} = \frac{5}{2} N_A k_B = \frac{5}{2} R \quad \text{—— (2)}$$

$$\therefore C_p = C_V + R = \frac{5}{2} R + R$$

$$C_p = \frac{7}{2} R \quad \text{—— (3)}$$

$$\gamma = \frac{C_p}{C_V} = \frac{7/2 R}{5/2 R}$$

$$\therefore \boxed{\gamma = \frac{7}{5}} \quad \text{—— (4)}$$

- For diatomic gas containing non rigid vibrating molecules internal energy per mole is

$$E = \frac{3}{2} N_A k_B T + \frac{2}{2} N_A k_B T + \frac{2}{2} N_A k_B T$$

$$E = \frac{7}{2} N_A k_B T \quad \text{—— (1)}$$

$$\therefore C_V = \frac{dE}{dT} = \frac{7}{2} N_A k_B = \frac{7}{2} R \quad \text{—— (2)}$$

$$C_p = C_V + R = \frac{7}{2} R + R$$

$$C_p = \frac{9}{2} R \quad \text{—— (3)}$$

$$\gamma = \frac{C_p}{C_V} = \frac{9/2 R}{7/2 R}$$

$$\therefore \boxed{\gamma = \frac{9}{7}} \quad \text{—— (4)}$$

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③ Polyatomic gases:- Gases which have molecules containing more than two atoms are polyatomic gases (ammonia-NH_3). Each molecule of polyatomic gas has 3-translational dof, 2 rotational dof & some polyatomic have 3 dof for rotation also. & 'f' no. of vibrational dof.

So internal energy per mole of polyatomic gas is

$$E = \frac{3}{2} N_A K_B T + \frac{3}{2} N_A K_B T + f \times \frac{2}{2} N_A K_B T.$$

$$E = (3+f) N_A K_B T \quad \text{—— (1)}$$

Molar sp. heat at constant volume is

$$C_V = \frac{dE}{dT} = (3+f) N_A K_B$$

$$C_V = (3+f) R \quad \text{—— (2)}$$

$$\therefore C_p = C_V + R = (3+f)R + R$$

$$\therefore C_p = (4+f) R \quad \text{—— (3)}$$

$$\gamma = \frac{C_p}{C_V} = \frac{(4+f)R}{(3+f)R}$$

$$\therefore \boxed{\gamma = \frac{4+f}{3+f}} \quad \text{—— (4)}$$

Q Define three coefficients of heat for a body and show that $a + r + t_r = 1$.

→ ① Coefficient of absorption (Absorptive power or Absorptivity) [a]

It is defined as the ratio of amount of heat (thermal energy) absorbed by a body to the total amount of heat incident upon it.

$$a = \frac{Q_a}{Q} \quad \text{—— (1)}$$

2) Coefficient of reflection (reflectance) [r]

It is defined as the ratio of amount of heat reflected by a body to the total amount of heat incident upon it.

$$r = \frac{Q_r}{Q} \quad \text{— (2)}$$

3) Coefficient of transmission (Transmittance) [t]

It is defined as the ratio of amount of heat transmitted by a body to the total amount of heat incident upon it.

$$t_r = \frac{Q_t}{Q} \quad \text{— (3)}$$

Let Q be the total amount of radiant heat incident on a body. Q_a, Q_r & Q_t be the amount of heat absorbed, reflected & transmitted by the body.

$$\therefore Q = Q_a + Q_r + Q_t \quad \text{— (4)}$$

Add (1), (2) & (3)

$$a + r + t_r = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q}$$

$$a + r + t_r = \frac{Q_a + Q_r + Q_t}{Q} \quad \text{— (5)}$$

put (4) in (5)

$$a + r + t_r = \frac{Q}{Q}$$

$$\therefore \boxed{a + r + t_r = 1} \quad \text{— (6)}$$

* Athermanous substance: It is a substance which do not transmit heat radiations incident upon it.

$\therefore t_r = 0, \quad a + r = 1$

eg: water, wood, copper, moist air, benzene etc.

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* Diathermanous substance:- It is a substance through which heat radiations can pass.
∴ $t_r \neq 0$
eg. Glass, quartz, Sodium chloride, hydrogen, oxygen, dry air etc.

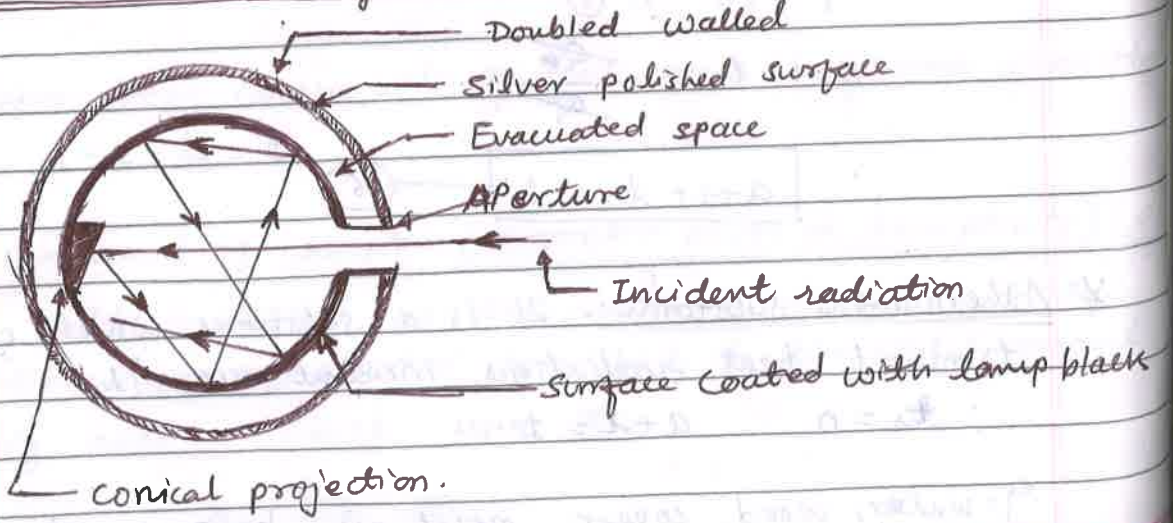
* Perfect transmitter:- It is a substance through which all the heat radiations can pass, which are incident upon it.
∴ $t_r = 1$, $a = 0$, $r = 0$

* Perfect reflector:- It is a substance which reflect all the heat radiations incident upon it.
∴ $r = 1$, $a = 0$, $t_r = 0$

* Perfect absorber (perfectly black body)
It is a substance which absorbs all the heat radiations incident upon it.
∴ $a = 1$, $r = 0$, $t_r = 0$

Lamp black or plantinum black absorb 97% of heat.

* Construction of Ferry's black body



It consist of a double walled hollow sphere having a small aperture through which radiant heat can enter. The space betⁿ the walls is evacuated and outer surface of the sphere is silvered while inner surface of inner sphere is coated with lamp black. There is a conical projection on the inner surface of sphere opposite the aperture. The projection ensures that a ray travelling along the axis of the aperture is not incident normally on the surface & is therefore not reflected back along the same path. So the radiant ray suffer multiple internal reflections inside the spheres. At each point of incidence nearly 97% of energy is absorbed. so after some time total energy is absorbed by the body. Hence it is called as perfectly black body.

* Emissive power (R): The emissive power of a body is defined as the amount of heat radiated per unit time per unit area at a given temp.

$$R = \frac{Q}{At}$$

Q = Amount of heat radiated
A = Surface area, t = Time for radiation of heat

SI unit of $R = \frac{J}{m^2 s}$ or W/m^2.

Dimension

$$[R] = [M^1 L^0 T^{-3}]$$

The amount of heat radiated by a body depends on.

1) The absolute temp of the body (T)
2) Nature of body - the material & nature of surface - (polished or not)
3) Surface area of the body (A)
4) Time duration for which body emits radiations (t)

* At 800°c - part of energy emitted is in the visible range & body looks red.
* At 3000°c - body looks white hot, The filament of tungsten lamp appears white hot

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* **Coefficient of emission or Emissivity (e)**: For a given surface

It is defined as the ratio of emissive power of a given surface to the emissive power of a perfect black surface, at the same temp.

$$e = \frac{R}{R_B}$$

R = Emissive power of a given surface
R_B = ——— " ——— perfect black surface at same temp.

* For perfect black body $\Rightarrow e = 1$.
* For perfect reflector $\Rightarrow e = 0$
* For ordinary body $\Rightarrow 0 < e < 1$
* Emissivity does not depends upon colour & composition of the surface in addition to the temp.
* For copper $\Rightarrow e = 0.3$
* Emissivity is larger for rough surfaces
* Emissivity is smaller for smooth & polished surfaces.
* Emissivity varies with temp & wavelength of radiations.

Q state Kirchhoff's law of heat radiations & describe its theoretical proof.

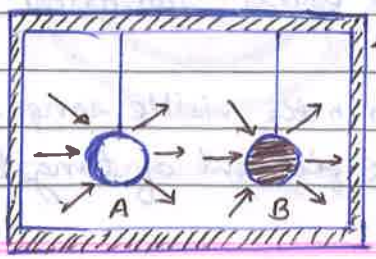
$\Rightarrow$ **Kirchhoff's law of thermal radiation:**

It states that coefficient of absorption is equal to the coefficient of emission of a body at the same temp.

OR→ At a given temp, the ratio of emissive power to coefficient of absorption of a body is equal to the emissive power of a perfect blackbody at the same temp. for all wavelengths.

$$a = e$$ at the same temp.

or $$a(\lambda) = e(\lambda)$$



← Constant temp enclosure
A = ordinary body
B = perfectly black body.

Consider an ordinary body A & perfect blackbody B of identical geometrical shapes. placed in a constant temp enclosure. In thermal equilibrium both bodies will be at same temp. as that of enclosure.

For the bodies.

R = Emissive power of body A

R_B = Emissive power of body B.

a = coefficient of absorption of body A.

If Q is the quantity of radiant heat incident on each body per unit time.

Q is the quantity of radiant heat absorbed by body B in unit time. & $Q_a = aQ$ is the quantity of radiant heat absorbed by body A in unit time.

As the temp of bodies A & B remains the same, ie both the bodies emits the same amount as they absorbe in unit time.

∴ Quantity of radiant heat absorbed by body A = Quantity of heat emitted by body A.

$$\therefore aQ = R \quad \text{—①}$$

similarly,

Quantity of radiant heat absorbed by body B = Quantity of heat emitted by body B.

$$\therefore Q = R_B \quad \text{—②}$$

Divide ① by ②

$$\therefore \frac{aQ}{Q} = \frac{R}{R_B}$$

$$\therefore a = \frac{R}{R_B} \quad \text{—③}$$

or $$\boxed{\frac{R}{a} = R_B} \quad \text{—④}$$

As $\frac{R}{R_B} = e$ —⑤ From eq ③ & ⑤

$$\boxed{a = e} \quad \text{—⑥}$$

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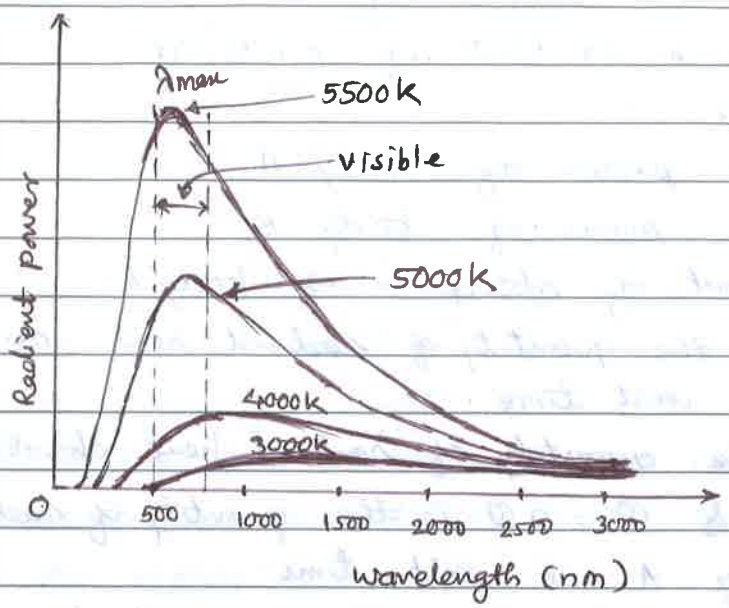
Q. Explain the spectral distribution of blackbody radiation. with neat graph of radient power against wavelength.

⟹



The energy distribution of blackbody radiations as a function of wavelength, as shown in graph.

Kept the source of radiation at diff. temp^s and measured the radient power, corresponding to wavelengths. The curves showing variation of radient power per unit area as a function of wavelength λ, at diff. constant temps. The experimental observations depends only on the absolute temp. T of a blackbody & was independent of the material. It is also observed that

1) At a given temp, the energy is not uniformly distributed in the spectrum of blackbody.
2) At a given temp. radient power emitted initially increases with increase of wavelength, reaches its max^m value & then decreases. The wavelength corresponding to max^m intensity (λ_{max}) is the characteristics of the temp. of the radiating body.
3) Area under the curve represents total energy emitted per unit area per unit time by the blackbody at all wavelengths
4) The peak of curves shifts towards the left (shorter wavelength) ie λ_{max} decreases with increase in temp.
5) At higher temp: the radient power (total energy emitted per unit time per unit area) ie area under the curve corresponding to all the wavelengths increases.
6) At a temp of 300K (Room temp.), the most intense of the waves has wavelengths of about 5×10^{-6} m. The radient power is smaller for wavelengths diff from this value.

* Wien's Displacement Law:

The wavelength (λ_{max}) for which emissive power of a blackbody is max^m; is inversely proportional to the absolute temp. of the blackbody.

$$\lambda_{max} \propto \frac{1}{T}$$

$$\boxed{\lambda_{max} = \frac{b}{T}}$$ where b is Wien's constant.

$b = 2.897 \times 10^{-3}$ mk

λ_{max} indicates the wavelength at which the blackbody dominantly radiates. So it corresponds to the dominant colour of radiating body and is function of temp.

* For white dwarfs (hot stars) - with surface temp ~ 10000 K.
* For red giants - they are cooler corresponding to surface temp ~ 3000 K.

This law is useful to determine temp^s of distant stars, Sun, moon etc.

* Stefan-Boltzmann Law of radiation:

The rate of emission of radiant energy per unit area of a perfect blackbody is directly proportional to the fourth power of its absolute temp.

$$R \propto T^4 \quad \text{i.e} \quad \frac{Q}{At} \propto T^4$$

$$\therefore R = \sigma T^4 \qquad \frac{Q}{At} = \sigma T^4.$$

where σ is Stefan's constant ($\sigma = 5.67 \times 10^{-8}$ J/m^{2}s K^4 or W/m^{2}K^4)

$$[\sigma] = [M^1 L^0 T^{-3} K^{-4}]$$

It shows that power radiated by a perfect blackbody depends only on its absolute temp. & not any other characteristics like colour, material, nature of surface etc.

For ordinary body (not blackbody) the energy radiated is less & is given by

$R = e\sigma T^4$ — e = emissivity of the surface.

If a perfect blackbody having absolute temp T is kept in a surrounding of lower absolute temp T_0.

The energy radiated by body per unit time per unit area $= \sigma T^4$. —①

The energy absorbed by body from surrounding per unit time per unit area $= \sigma T_0^4$ —②

∴ Net loss of heat by body per unit time per unit area $= \sigma T^4 - \sigma T_0^4$.

for an ordinary body, $= \sigma(T^4 - T_0^4)$ —③

Net loss of heat per unit time per unit area $= e\sigma(T^4 - T_0^4)$ —④

If $T < T_0$.

∴ Net gain of heat by body per unit time per unit area $= e\sigma(T^4 - T_0^4)$ —⑤

* If absolute temp of body is doubled, the power radiated will increase by a factor $2^4 = 16$.

* If a body radiates with some rate at room temp. (300 K, or 27°C) The rate will double even if we increase the temp of body by 57°C.